Frobenius Theorem

Exercise 1. Let $X : (x, y, z) \mapsto \frac{\partial}{\partial x} - y\frac{\partial}{\partial z}$ and $Y : (x, y, z) \mapsto \frac{\partial}{\partial y}$ be two vectors fields on $\mathbb{R}^3$.

1. Prove that $\forall p \in \mathbb{R}^3$, $(X(p), Y(p))$ is linearly independent.
2. Compute $[X, Y]$. Is the distribution spanned by X and Y integrable?
3. Recover this result without using Frobenius Theorem.

Exercise 2. Let $X : (x, y) \mapsto x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$ et $Y : (x, y) \mapsto x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$ be two vector fields on $\mathbb{R}^2$.

1. Compute $[X, Y]$. Are there coordinates (s, t) on some neighborhood of $(1, 0)$ such that $X = \frac{\partial}{\partial s}$ and $Y = \frac{\partial}{\partial t}$?
2. Compute the flows of X and Y.
3. Build explicit coordinates (s, t) on some neighborhood of $(1, 0)$ such that $X = \frac{\partial}{\partial s}$ and $Y = \frac{\partial}{\partial t}$.