Vector fields

Exercise 1 (Flow box)**.** Let M be a compact manifold and let X be a vector field on M. Let $p \in M$ be such that $X(p) \neq 0$, prove that there exist local coordinates around p such that $X = \frac{\partial}{\partial x_1}$.

Are the orbits of X submanifolds of M?

Exercise 2 (Transitivity of the group of diffeomorphisms)**.**

1. Let a and b in the open ball $B = \{x \in \mathbb{R}^n \mid \|x\| < 1\}$. Prove that there exists a diffeomorphism $f : \mathbb{R}^n \to \mathbb{R}^n$ such that $f(a) = b$ and $f = \mathrm{id}$ on $\mathbb{R}^n \setminus B$.

2. Let M be a connected manifold, prove that it is path connected.

3. If moreover $\dim(M) \geqslant 2$, prove that the natural action of the group of diffeomorphisms of M is k-transitive for any $k \in \mathbb{N}^*$.

4. What happens if $\dim(M) = 1$?